

ΣΟΦΙΑ—SOPHIA

Game as didactic strategy to develop numerical thought in the four basic

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* This article is the result of the research project: "The game as a didactic strategy to develop numerical thinking in the four basic operations", developed by three teachers who belong to the Mathematical Education Research Group de la Universidad del Quindío (GEMAUQ, for its initials in Spanish)



Abstract

This article is the result of a research carried out by teachers who belong to the Research Group in Mathematical Education of the Universidad del Quindío (GEMAUQ, for its initials in Spanish), in which it was sought to develop different skills and relationships to familiarize and reinforce the four basic operations (addition, subtraction, multiplication and division) in fifth grade students, assuming that games occupy a primordial place among the multiple activities of children. The didactic strategy consisted of working on a series of activities and/or games in each one of the mathematical operations and the combination of these, as well as in problem solving, whose implementation allowed to generate greater motivation and interest in students in the proposed topic. Once again, it is ratified that teaching mathematics using games as a didactic strategy to replace conventional didactic methods applied in the classroom transforms the teaching-learning process and how teachers and students access to knowledge in the four basic operations of numerical thinking.

Keywords: Mathematical games, didactic strategy, numerical thinking, basic operations, mathematical education

Introduction

Games occupy a primordial place among the multiple activities of children, and in their socio-affective development, in agreement with [Aristizábal, J; Colorado, H and Alvarez, D.](#) (2011). They state that “play as a didactic strategy and as a playful activity in the integral development of children is relevant in the learning of mathematics, as it can act as a mediator between a concrete problem and abstract mathematics, depending on the intentionality and type of activity” (p.2) For this reason, the project “Playing as a didactic strategy to develop numerical thinking in the four basic operations” allows to develop different calculation skills and relationships, to be familiar with and reaffirm the knowledge of the basic operations: addition, subtraction, multiplication and division through surprise, practice and fun, as well as it serves as an alternative evaluation of those operations for its dynamism, in fifth grade students. The game as a teaching learning strategy has shown significant results in students for learning mathematics; which is evidenced in some investigations such as: “Research on games, interaction and construction of mathematical knowledge”, by Eduardo Mercè and Deulofeu, Jordi of the Department of Didactics of Mathematics and Experimental Sciences of the Autonomous University of Barcelona; “Eduardo Mercè and Deulofeu: practical applications for the children’s classroom “, by Ruesga Ramos, María del Pilar.

Current education requires people with critical, analytical, reflexive ability; and this is achieved through the development of thought. A person with a high intellectual development is able to interpret, argue, propose, pose and solve problems in different contexts; therefore, for the acquisition of numerical sense, it is necessary to provide children, through

play, with rich, varied and meaningful situations that stimulate intelligence and imagination, as it is set by the curricular standards “... activities focused on understanding the use and meanings of numbers and numbering, understanding the meaning of operations and relationships between numbers, and the development of different techniques of calculation and estimation. (P.58).

Therefore, teachers today have the challenge of reframing their pedagogical practices where they seek to have their students take ownership of the concepts and understand the importance of mathematics. In this regard, López (2005) states that: “it is necessary to look for alternative ways to present the contents from situations and activities that represent a significant meaning for the student; these will allow students to generate conjectures, analyze them with their peers and consciously put into play the previously acquired knowledge.”

Thinking involves different mental actions that evolve when learners modify their cognitive structures, as the mathematical game in its dynamics puts into action the ability to reason, propose, communicate the mathematical form from orality and writing; that is, when it appropriates of language, history, the meaning of mathematical concepts and how they involve other concepts that are simultaneously developed in loops, generating ever more solid knowledge. This process makes students the main protagonists of their learning. Professor [Paulino Murillo](#) (2003) affirms that students must build their own learning, they are autonomous and integrate their experiences to others already known, so that they do not continue in the search of the development of memory and repetition, and it is precisely in this subject where it is recognized the acquired knowledge.

This work was financed by the Universidad del Quindío (Armenia, Colombia); it was performed in the educational institution Henry Marín Granada of Circasia, with fifth graders; this research allowed the development of numerical thinking through the intervention of the game as a strategy to master the four basic operations, which was evidenced by the capacity of relation, analysis, comprehension, processes, abstraction, synthesis, generalization and the development of processes that involve the basic operations.

Materials and methods

The main objective of the research was to design and put into operation a didactic strategy based on games to strengthen numerical thinking in the four basic operations, in fifth grade students, with participation of the academic and pedagogical community. In order to do this, an experimental research was carried out; in addition, it was exploratory, as it developed a didactic strategy that served to solve the problems encountered in fifth grade students when dealing with basic operations, allowing them a clearer understanding of the subject.

The study population consisted of two groups of fifth grade students of the school Henry Marín Granada of the municipality of Circasia in the department of Quindío, with whom we worked according to the following experimental design.

Experimental design

It was chosen the experimental design pretest-posttest, taking one of the groups as the experimental group and the other as a control group; it is to notice that the groups were nonequivalent because they had similar characteristics as they belonged to the same institution, the same degree, an equivalent social stratum, and there were no major oscillations between their ages.

This design is called “pretest - posttest design with a non-equivalent control group” by [Campbell and Stanley \(1975\)](#), quoted by [Hernandez, Fernandez and Batista \(2010\)](#); it has the following structure:

The interrupted line indicates that the experimental (E) and control (C) groups have not been taken at random.

O1 and O3: These are the measurements (taken) before

O2 and O4: Are the measurements (taken) after

X: Treatment

In the research, the following hypothesis is held: the development of numerical thinking in the four basic operations is greater with the use of a didactic strategy through the game than when using a traditional strategy.

Manipulation of the research variable

The variable to be manipulated was the independent one: development of numerical thinking in the four operations with the conditions of didactic strategy and traditional strategy. Children in the two groups, experimental and control, respond to *pretest* (pre-treatment measurements) and *post-test* (post-treatment measurements).

Treatment

Students in the experimental group received treatment with the didactic sequence based on mathematical activities and games, as well as problem solving. It is important to emphasize that with the application of the didactic strategy, mental operations are strengthened, which according to [L. Alonso \(2000\)](#) are:

- Receptive: Observe, identify and listen, identify elements in a set
- Comprehensive: analytic (analytical thinking), compare/relate, sort/classify, to abstract, problem solving (complex thinking), deduce/infer, compare/experiment, analyze perspectives/to interpret, to transfer/to generalize, to analyze/to link, to understand/to conceptualize
- Symbolic, expressive: to represent (textual, graphic, oral ...)/communicate.

Students in the control group were treated with traditional methodology. According to [Flórez \(1994\)](#), the basic method of learning in the traditional model is the academic and verbalist one, in which classes are taught to students under a regime of discipline.

Applied didactic strategy

1. Fifth grade teachers of the Henry Marín Granada institution in the municipality of Circasia were interviewed about the kind of methodology they use for teaching the four mathematical operations (addition, subtraction, multiplication and division).
2. From the fifth grade students of the institution, two groups were randomly selected as a sample; the experimental group and the control group.
3. The pretest was applied to both the control and the experimental group to know the students' previous concepts about the basic operations, which the group of researchers designed, validated and applied to both groups; this pretest consists of a 16-item questionnaire, of which 15 were focused on a problematic situation where one or several operations were involved, whose response was multiple choice with four options of answer, but with only one answer; the remaining

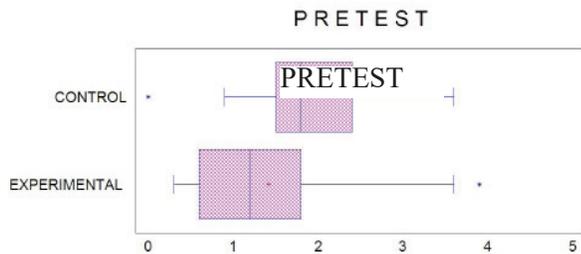
question consisted in exercises to solve mathematical operations of algorithmic form. The purpose of the pretest was to determine the reasoning used by students in solving a problem situation, and to investigate previous concepts and algorithms they had about addition, subtraction, multiplication and division.

4. The conditions of homogeneity were established, for which the pretest questionnaires were scored, the results were processed in the statistical package *Statistic*, and the results were analyzed.

The pretest analyses of the control and experimental groups are presented below.

Homogeneity test

Figure 1. Analysis of the pretest results



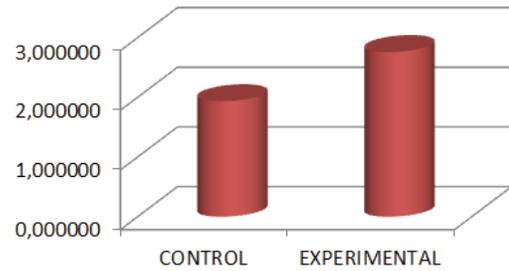
Source: self-made

In the box diagram, it can be observed the homogeneity of the pretest results. 50% of students in the control group scored below 1.8; and 50% of students in the experimental group reached scores below 1.2.

Figure 2. Comparison of mean values in the *pretest*

Source: self-made

In the Student's t-test performed to compare the results of the pretest of the control group and the experimental group, it is observed that at 95% confidence level there is no significant difference between the means of the groups; that is, the groups are homogeneous. As it is evident with the P-value ($P\text{-value} = 0.124028 > 0.05$). This shows that both the control and the experimental group obtained very similar results, indicating that they have the same level of knowledge of the subject.



5. In the control group, it was developed the theme of basic operations, following the traditional model proposed by teachers who dictate mathematics to students in fifth grade, which was consulted in the previous interview.

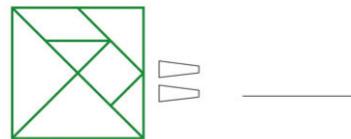
6. In the experimental group, it was implemented the game as a didactic strategy to develop numerical thinking in the four basic operations, which consisted of working on a series of activities and/or games, which caused interest and favored the participation of students in each one of the stages of the game, using mathematical operations (addition, subtraction, multiplication, division and the combination of these), as well as in problem solving; applied to the experimental group. Some of the activities and/or games were made in two sessions due to the dynamics of the game itself. Below are some of the activities that were used in the development of the didactic strategy.

Figure 3. Activity of the strategy using the tangram

Assume that the smaller triangle is worth less than 1



How much is it the whole tangram?



How much is it each one of the following figures?

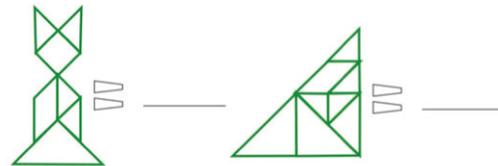
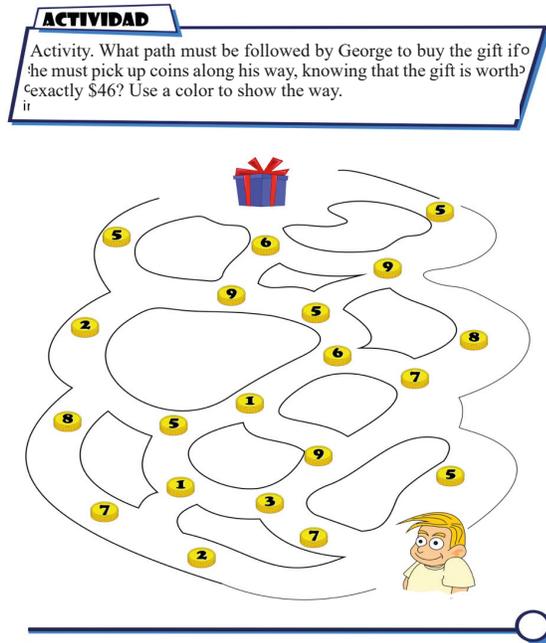
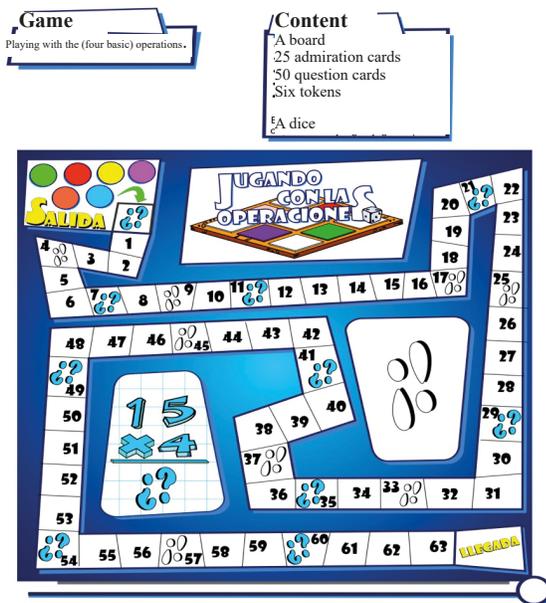


Figure 4. Activity of the strategy using written activities



Source: self-made

Figure 5. Activity of the strategy using a designed game.



The board can be found in the cd, along the route:
 games/playing operations
 Departure
 Playing with the operations
 Arrival

Source: self-made

7. During the course of the investigative process, continuous monitoring was carried out, a constant monitoring of the sample groups to determine the students' attitudes towards the methodologies; in most cases this monitoring was obtained through observation of competitors.

8. After completing the strategy, a *posttest* (which consisted of the same *pretest* questionnaire) was applied to the groups that made up the sample.

Below are the calculations of the arithmetic mean, median and standard deviation of the *pretest* and *posttest* groups.

Table 1. Comparison of the arithmetic mean, median and standard deviation of the *pretest* and *posttest* groups

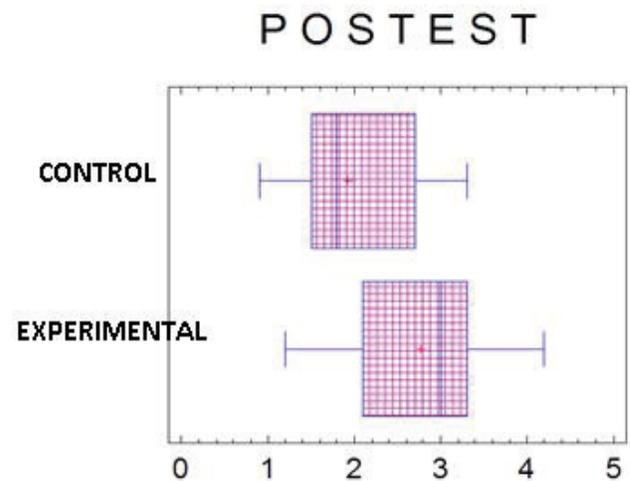
Group	Mean		Median		Standard deviation	
	Pretest	Posttest	Pretest	Posttest	Pretest	Posttest
Control	1,8833	1,93333	1,8	1,8	0,82836	0,79187
Experimental	1,4143	2,75714	1,2	3	1,00463	0,951615

Source: self-made

Results

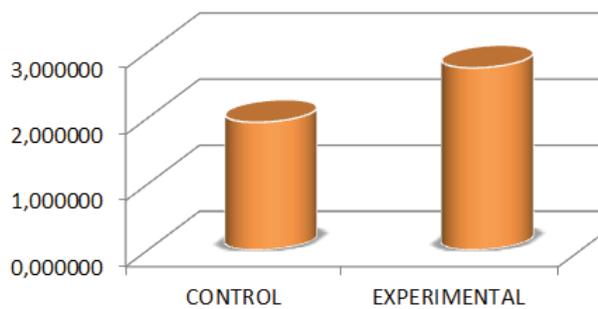
Techniques of statistical analysis of results: According to the experimental design, the Student's t-test was used to establish the differences between the compared groups, of which it is shown the posttest analysis of the control and experimental groups.

Figure 6. Analysis of posttest results



Source: self-made

Figure 7. Comparison of the arithmetic mean of *posttest* groups



Source: self-made

Tabulation and interpretation of the obtained results: when comparing the mean, median and standard deviation between *pretest* and *posttest* groups (Table 1), it is evident a greater efficiency in the experimental group when using the game strategy, in relation with the control group in which the traditional methodology was used by the teachers (Figure 6); in the box diagram, the heterogeneity of *posttest* results can be observed. Fifty percent of the students in the control group scored below 1.8 and 50% of the students in the experimental group obtained scores below 3.0.

In the Student's t-test performed to compare the *posttest* results of the control group and the experimental group (Figure 7), it was observed that at a 95% confidence level, there was a significant difference between the groups' averages. (P-value = 0.00611012 < 0.05)

Discussion of results

Psychologists emphasize the importance of play in childhood as a means to form personality and to learn experimentally to relate in society, to solve problems and conflictive situations. In the work carried out, it was possible to see that the application of the different games developed in the strategy allowed students to integrate, interact, lead, confront ideas and generate strategies to solve the problems or challenges posed in the games; all this allowed appropriation of concepts and development of numerical thinking. According to [Ogalde and Bardavid \(1997\)](#), the advantages provided by didactic materials make them indispensable instruments in academic formation: They provide information and guide learning, that is, they provide a concrete basis for conceptual thinking and contribute to the increase of meanings; they develop continuity of thought, make learning more durable, and provide a real experience that stimulates student activity.

The five general processes that are established in the *Curriculum guidelines of mathematics* are: to

formulate and solve problems; to model processes and phenomena of reality; to communicate; to reason and compare; and to exercise procedures and algorithms. In the different moments in the development of the strategy, spaces were generated where the students through the game could approach the algorithms of the operations with natural numbers, improving their ability of mental calculation.

Subsequently, we worked on situations-problems that were solved through games, identifying which operations should be taken into account to find such solutions. In this regard, [Godino \(2004\)](#) states that through the resolution of mathematical problems, students must acquire adequate ways of thinking, habits of persistence, curiosity and confidence in unfamiliar situations that will be useful outside the math classroom.

[Jean Piaget \(1978\)](#) also studied the transition of adolescents' reasoning from what he called "concrete operative thinking" to "formal operative," and proposed a set of logical-mathematical operations that could explain this step. When students manipulated the different games proposed in the strategy, a concrete thought was evident, which after several sessions allowed some students to transition from concrete thinking to formal thinking.

On the other hand, [Miguel de Guzmán \(1992\)](#) points out that, beyond the traditional branches of mathematics, arithmetic and geometry, in its historical evolution the mathematical spirit would have to face the following aspects:

- The complexity of the symbol.
- The complexity of change and deterministic causality.
- The complexity arising from the uncertainty in uncontrollable multiple causality.
- The complexity of the formal structure of thought.

Here, one can see a clear relation with the five types of mathematical thought enunciated in the *Curricular Guidelines*: in arithmetic, numerical thought; geometry, spatial and metric thinking; in algebra and calculus, metrical and variational thinking, probability and statistics, as well as random thinking. In the didactic strategy applied in the project, the numerical thinking was developed through the game, in the four basic operations supported in the other thoughts, which allows to generate new investigations in the other thoughts through the game as didactic strategy.

In order to validate the appropriation of the concepts, we worked with problem situations whose solution

required the basic operations (addition, subtraction, multiplication and division), with which it was shown the applicability of the subjects seen and the logical sequence to arrive at the solution of a problem. According to Vergnaud (1995): problem-solving teaching emphasizes thought processes, processes of learning and taking mathematical contents, being important that students manipulate mathematical objects, activate their own mental capacity, and exercise their creativity, reflect on their own thought process, make transfer of activities or other aspects of their mental work, gain self-confidence, have fun with their own mental activity, prepare themselves for other problems of science and everyday life, and for the new challenges of technology and science.

It was evidenced that low reading comprehension in both groups was a determining factor in the difficulties for problem solving, so it is of vital importance that teachers provide students with the necessary tools to improve understanding and interpretation of problem situations. Parra (1990) states that:

Problem solving analysis is limited to intentionally structured learning situations and linked to some field of study, such as those given in the school dynamics, that having the elements to understand the situation described by the problem supposes that the subject who must solve the problem has had access to or has constructed that declarative knowledge and the correspondent procedural knowledge, (both of) which are required as a minimum necessary to understand information, establish relationships and use procedures in order to solve the problem that has been raised.

Conclusions

The general objective was fulfilled through the implementation of the didactic strategy from the game, since it allowed numerical thinking to be strengthened in the four basic operations in the fifth grade students of the Henry Marín Granada Educational Institution of the municipality of Circasia, in the Department of Quindío. The enrolled groups had homogeneous conditions, that is to say, they had the same characteristics in terms of the theoretical assumptions that they have at the beginning of the execution of the investigation project, which allowed greater efficiency in the tests and activities that were done to reach the proposed objectives.

The implementation of the game allowed to generate greater motivation and interest in the students for the proposed theme. The working hypothesis was verified, because significant differences were evidenced in the pretest and posttest scores of both control and experimental groups.

In a didactic (activity) as the one developed in this project, mathematics acquires a new meaning for the student. In addition to the motivation and good attitude that students showed when working with games, other important aspects are highlighted to the procedures simply algorithms such as those related to giving results and without the argument.

Group games were more welcomed by the students because they allowed to generate competition between them. In these moments of change, it is necessary to reflect on the teaching of mathematics regarding the conventional didactic methods used in the classroom, and to try to transform the teaching-learning process and the way in which teachers and students access to knowledge.

Recommendations

Taking into account the educational reality, it is recommended that teachers present and accept innovative pedagogical and didactic strategies within the framework of the game as a teaching strategy, which lead to the development of mathematical thinking. It is suggested to continue the proposal of the game as a didactic strategy to develop numerical thinking in the four basic operations and in other topics, as an effective strategy to overcome the difficulties found in mathematics education.

It is suggested to teachers in the area of basic education mathematics the application of strategies aimed at developing mathematical thinking in students, in order to enhance the skills that enable them to improve access to knowledge.

The activities developed in this project show the arduous process involved in developing the proposed games and activities, but this effort will be in vain to the extent that institutions and teachers do not involve these materials in their classrooms.

Faced with this situation, the university should become an articulating axis for the construction of educational materials for institutions of basic and intermediate education. This will allow in the long term the development in the region of its own strategies.

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