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# Visualization Processes in Solving Math Problems in <br> Elementary School supported by ICT-Mediated Learning Environments 

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Processos de visualização na solução de problemas de matemática no nível primário, suportados por ambientes de aprendizado mediados pelas TIC

> Heiller Gutiérrez Zuluaga * Jorge Hernán Aristizabal Zapata ** Julián Andrés Rincón Penagos * * *

*Master in Education. Head Teacher of the Degree in Mathematics - Universidad del Quindío. Armenia, Quindío, Colombia.<br>hgutierrez@uniquindio.edu.co https://orcid.org/0000-0003-2057-5859<br>**Master in Education. Professor of the Degree in Mathematics - Universidad del Quindío. Armenia, Quindío, Colombia. jhaz@uniquindio.edu.co. https://orcid.org/0000-0002-8875-1823<br>***Master in Education. Affiliation: Professor of the Degree in Mathematics - Universidad del Quindío. Armenia, Quindío, Colombia. jarincon@uniquindio.edu.co


#### Abstract

In the department of Quindio, it is evident in the results of the 3rd and 5th grade Saber Tests that students have various difficulties in mathematics, one of the causes is to face problem solving. This is because many teachers sometimes give little importance to processes such as visualization, which contributes to the development of a deeper and more meaningful understanding of both mathematical ideas, as well as the relationships between mathematical concepts. In other cases, they are unaware of the potential of learning environments and the integration of ICTs, which transcend the physical classroom, enriching it with new pedagogical alternatives and giving students meaningful experiences and better learning opportunities. This research was developed under a qualitative interpretive methodology, whose objective was to analyze incidences of visualization processes supported with ICT environments in the resolution of mathematics problems by students of basic primary education. It was found through the use of educational software that visualization plays a very important role in the problem-solving styles of students according to the categorization of the proposed tasks.


Keywords: Visualization processes, math problem solving, ICT-Mediated Learning Environments, educational software.

## Resumo

No departamento de Quindío, é evidenciado nos resultados dos testes do Saber de $3^{a}$ e $5^{a}$ séries que os alunos apresentam várias dificuldades em matemática, uma das causas é enfrentar a resolução de problemas. Isso ocorre porque muitos professores às vezes dão pouca importância a processos como a visualização, o que con- tribui para o desenvolvimento de um entendimento mais profundo e significativo das idéias matemáticas e das relações entre os conceitos matemáticos. Ou, em outros casos, eles não conhecem o potencial dos ambientes de aprendizagem e a integração das TICs, que transcendem a sala de aula física, enriquecendo-a com novas alternativas pedagógicas e proporcionando aos alunos experiências significativas e melhores oportunidades de aprendizagem. A pesquisa foi realizada sob uma metodologia qualitativa de tipo interpretativo, cujo objetivo foi analisar incidentes dos processos de visualização suportados em ambientes de TIC na resolução de problemas de matemática por estudantes do ensino fundamental. Foi constatado através do uso do software educacional que a visualização desempenha um papel muito importante em termos de estilos de resolução de problemas dos alunos, de acordo com a categorização das tarefas propostas.

Palavras-chave: Processos de visualização, solução de problemas de matemática, ambientes de aprendizado mediados pelas TIC. Software educacional.

## Introduction

Mathematics Education is an area of research with relevant theoretical and practical results for educational problems in favor of the integral development of students and society. The Saber tests, as an indicator of verification of the competencies acquired by students, show low academic performance in the area of mathematics, which becomes a great challenge for both educators in educational institutions as mentioned by Aristizábal, Colorado, \& Gutiérrez (2016: p. 118). Today's teachers have the challenge of resignifying their pedagogical practices where they seek that their students appropriate the concepts and understand the importance of mathematics. As for researchers in mathematics education, they seek to interpret the meaning that the teaching and learning of mathematics have for the participants, by living within the classroom, participating or not in the instructional process (Kilpatrick, 1998: p. 5). What is alarming is that Quindio does not appear among the departments with the best scores according to the results of the Saber test in recent years.

These results could change with the incorporation of information and communication technologies (ICT) in the classroom, as stated by Jaramillo and Castañeda (2009, p.160): The teacher creates the necessary conditions so that the student can learn directly in front of to the stimuli of the learning environment. An appropriate use of these, allows the teacher of Mathematics to be offered the opportunity to create enriched learning environments so that students perceive it as an experimental science and a significant exploratory process within their training. Fact stated by Ordaz (2002) when stating that

The use of technology can significantly improve learning, as it focuses on virtual manipulatives that help students increase
their ability to acquire skills and concepts, by offering a physical, mobile, buildable and disassemblable representation, which allows to visualize mathematical concepts in a concrete way (p.2).

On the other hand, visualization processes play a very important role in learning mathematics and in solving problems, as stated by Arcavi (2003 cited by Fernández, 2013: p. 21): visualization at the service of Problem solving can also go beyond its procedural role and inspire a general and creative solution. Likewise, representations of visual forms can be legitimate elements in mathematical demostration, in addition to contributing to a meaningful understanding of mathematical ideas and the relationships between mathematical concepts (Zimmermann and Cunningham, 1991).

Despite the government's efforts to provide educational institutions with technological resources such as computers to educate and tablets, many of the teachers who guide elementary school do not have the necessary training to articulate the use of ICT with mathematics , ignoring what Akkufl (2000: p.25) states: The computational capacity of technological tools broadens the range of problems accessible to students and also allows them to execute routine procedures quickly and accurately, which gives them more time to conceptualize and model. Especially, with the resolution of problems by privileging the visualization processes whose function, among others, is to support or guide the development of a problem posed or allow the understanding of the deployment of a given procedure (Marmolejo \& González, 2013: p. $90)$. This is the reason why this investigation was framed in the incidence that visualization processes have in solving math problems in learning environments, through the use of ICT with students of Elementary Education who belong to the official sector of the Department of Quindío.

Our research took place in two moments. In the first, different activities were carried out with tangible material for this case: logic blocks and the barriers game (figure 1); in the second, four educational programs were implemented: Sokoban, The Clock, Machines and Angles ${ }^{1}$ (figure 2). Both with the purpose of identifying the visualization processes that students privileged while facing some proposed math problems. By encouraging elementary school students to use tangible and digital materials to teach different concepts in mathematics, their learning is enhanced, just as Escrivà, Jaime \& Gutiérrez (2018: 42) suggest: The teaching of mathematics in Primary Education includes a permanent use of visual representations of concepts, relationships and operations, through manipulative materials and drawings, diagrams, etc., both real and digital.

Figure 1. Barriers game and logic blocks.


[^0]Figure 2. Sokoban, Machines, Angles and The Clock.


On the other hand, it was investigated with the teachers who direct the school grades in which the research was carried out, the way in which they develop the class, the theoretical conception regarding the teaching of mathematics and the approach to problem solving, didactic resources used and assessment strategies that they apply with their students.

## Materials and Methods

The research process was framed in a quasiexperimental design, so it took three institutions with similar characteristics (Bisquerra, \& Alzina, 2009: p. 188). This type of design occurs due to the impossibility of randomizing the subjects to the experimental and control groups, or the fact of not being able to have a control group because it is not convenient or it is too expensive. The methodology was of a qualitative type,
oriented to comprehension, because according to Dorio, Sabariego and Massot (2009), it focuses on describing and interpreting an educational reality from within the subjects. Also, it is an action research (Latorre, 2009), as it is a plan to execute, an action or intervention of the researchers, an observation oriented to the collection and analysis of data, and a final reflection of the results obtained. The observations were made on 115 students, in the third ( 62 students) and fifth (53 students) grades, from three educational institutions in the Department of Quindío. Four methodological phases were carried out.

## Selected Educational Institutions

The selected educational institutions belonging to two municipalities in the Department of Quindío (Colombia) were:

- Institución Educativa Libre of Circasia: Two topics were worked on in this institution

Management of proximate time: A series of problems which students had to solve in order to observe the incidence of visualization were implemented through the educational program "The Clock". The problems raised were: setting the clock, synchronizing, indicating, writing and doing addition and subtraction between hours.

Angles: This concept was developed through the educational program "Angles". The following are the activities developed with the educational program: Construct and identify angles, locate them, estimate them in the solar system, represent them by positions, indicate their inclination, and measure the angle formed by the hands of the clock.

- Hojas Anchas Educational Institution²: In the first part, activities were carried out with the tangible material Logical Blocks and later we worked with the educational program Logical Relations.In this, four activities were carried out: Identify the figure, direct relationship, inverse relationship,
and find relationship. For each of the activities, different exercises were prepared, according to each of the levels of the program.
- Jesús María Morales Educational Institution of the municipality of Calarcá: In the first part, it was proposed to work with the game Barriers as manipulative material. Subsequently, the educational program Sokobanwas selected. The objective of this game is to take the boxes marked with an $X$ from an initial position to the final destination marked with red.


## Methodological Phases

Diagnostic Phase. In this phase, activities were developed aimed at knowing the problemsolving styles used by children and the way the teacher develops the class, the teacher's theoretical conception regarding the teaching of mathematics, the didactic resources used and the evaluation process applied to students.

Planning Phase. A set of activities was proposed with the purpose of providing feedback and transforming the pedagogical practice developed by teachers, in terms of identifying the incidence of visualization processes using an educational program at the time that students solve math problems.

For this phase, the following activities were considered (among others):
-Construction of the base of topics and problems to be implemented in the research process.

- Design and selection of the educational program that supports problem solving.
- Training of teachers and the work team.

Execution (field work). It included the activities carried out in the classroom and focused on identifying the impact that visualization processes have in solving problems by students when implementing the designed

[^1]computer programs. Similarly, in this phase, significant changes introduced by technology were observed to support visualization processes in solving math problems.

Evaluation Phase. In this, with the evidence collected in field work and according to the performance of the participating students,

The analysis and interpretation of the information obtained was carried out, contrasting it with the epistemological foundations of visualization.

## Results

According to the proposed methodology and the visualization tasks studied by theorists such as Gonzato, Fernández \& Díaz (2011), a categorization was made of the tasks that can be privileged when working with educational software, which are to do graphs or sketches, constructions, synoptic diagrams, transformations, counting, folding, views, compositions, gestures, description from the language, demonstrations without words and using representation systems. For fieldwork, workshops were held in the three educational institutions with third and fifth grade students, in two of which, Hojas Anchas and Jesús Maria Morales, activities were developed with and without software. Only in the Libre Educational Institution we worked with it.

The work that was carried out without educational software consisted of tasks with manipulable material (Logical Blocks) aimed at developing visualization processes. In this regard, Álvarez (2009: p. 2) affirms that the teaching of mathematics begins with an exploratory stage, which requires the manipulation of concrete material, and continues with activities that facilitate conceptual development based on the experiences collected by the students during the exploration. In addition, in the different activities designed for working with students in educational institutions, problem solving supported by the proposals of George Polya (1965) was considered.

Alan Schoenfeld (1992) on problem solving in mathematics. Table 1 presents a summary of the visualization skills that students privilege according to the implemented programs.

Table 1. Contrast of visualization skills using computer programs

| Program implemented |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Visualization skills | Sokoban | Clock | Angles | Machines |
| Making graphs or outlines |  | X |  |  |
| Implementing constructions | X |  |  | X |
| Making synoptic outlines |  |  |  |  |
| Implementing transformations |  | X |  | X |
| Counting | X | X | X |  |
| Making foldings |  |  |  |  |
| Views, | X |  |  |  |
| Making compositions | X |  | X |  |
| Gestures |  |  | X | $x$ |
| Description from language | X | X |  | X |
| Demonstrations |  | X | X | X |
| Without Words |  |  |  |  |
| Representation |  | X |  | X |
| Systems |  |  |  |  |

Next, the activities carried out, the processes that the students prefer and the results obtained in the educational institution Hojas Anchas with the program "Logical Relations" are described (figure 3).

Activity 1. Identify the figure

- Making Constructions: Students were given one or more conditions and they had to find the figure that met these conditions, for which they had to make the mental representation.
- Gestures: Students show different gestures accompanied by body expressions during and after facing the solution of each activity. These gestures can be performed unconsciously, which indicates the emotional state of the
students in solving problems with the educational program.
- Description from the language: It was observed that, when asked for more than one condition, students always repeated these aloud until they found the figures. In addition, they expressed aloud what the activity asked of them.
- Demonstrations without words: With their hands, students tried to form or draw the figure that was being asked of them, taking into account the different graphic codes.
- Using representation systems: From their previous concepts of geometric objects, students, according to the different activities proposed in the program, made mental constructions using their geometric representations such as squares or triangles and in this way they articulated them with size and color.


## Activity 2. Direct relation

- Gestures: The gesture that was observed was to form figures, since the students use their hands to represent the different figures (circle, triangle).
- Description from the language: Students express the transformations they must carry out aloud, repeating them constantly until they are clear about the solution to each activity.
- Representation systems: From their previous concepts of geometric objects, the students, according to the different activities proposed in the software, made mental constructions using their geometric representations such as squares or triangles and in this way they articulated it with size and colour.
- Demonstrations without words: students use the movement of their hands more or mentally perform the different transformations.

Activity 3. Inverse ratio

- Gestures: it was observed that the most common gesture was the movement of the hands when trying to understand or express the solutions of the activities.
- Description from the language: Students use more the movement of their hands or mentally perform the different transformations.
- Representation systems: From their previous concepts of geometric objects, the students, according to the different activities proposed in the software, made mental constructions using their geometric representations such as squares or triangles and in this way they articulated it with size and color.
- Demonstrations without words: students use the movement of their hands more or mentally perform the different transformations.


## Activity 4. Find the relationship

- Making Constructions: Students were given two figures, one in the entry window and the other in the exit window, then they had to build the transformation that allowed them to pass from one side to the other.
- Making transformations: In this activity the students had to identify, for example, a large red circle to a small yellow circle.
- Gestures: Finger pointing: Students point to the different options until they choose the transformation they consider correct. Hand rubbing: while developing or thinking about the solution to an activity.
- Representation systems: From their previous concepts of geometric objects, the students, according to the different activities proposed in the software, made mental constructions using their geometric
representations such as squares or triangles and in this way they articulated it with size and color.
- Demonstrations without words: students use the movement of their hands more or mentally perform the different transformations.

Figure 3. Logical construction of the response


Own source

The Jesús Maria Morales de Calarcá Educational Institution worked with the Sokoban program. The activities carried out, the processes that the students favor and the results obtained are described below.

- Views: They are used in terms of the direction that the boxes can take to move.
- Description from the language: students use expressions like "right, left, up, down stays in the corner, cannot move to either side".
- Making constructions: Students with their partner propose a strategy to solve the game.
- Making compositions: with the two partners, ideas are brought together to carry out the strategy to be solved.
- Counting: Students determine the time remaining to finish. They are also returned to see what has gone wrong and correct it.

In the Libre Educational Institution of Circasia we worked with two programs.

With the Angles program, they developed seven activities. Next, the activities carried out, the processes that students privilege and the results obtained are described as evidenced in image 4.

## Activity 1. Building angles

- Counting: At the beginning, a student wanted to use counting, since he had not observed that the screen showed the number of angles.
- Gestures: Students used a very particular facial expression when they did not find exactly the measure requested in the instructions.
- Demonstration without words: Students know that to draw angles the non-fixed segment had to be moved and a fixed segment remained.

Activity 2. Identifying angles

- Compositions: to identify the types of angles, in the case of the acute angle, the children know that they are less than $90^{\circ}$ and in turn right, they use this transformation from numerical to spatial to indicate angles with this description.
- Gestures: Both hands were used to show that an acute angle is more closed than an obtuse angle and to show a flat or null angle. Pencils were also used to represent.
- Demonstration without words: students discuss how they identified the types of angles by describing them.

Activity 3. Locating on the Radar.

- Counting: students used the radar lines to estimate angle measurements, they used the longest lines to count angles five by five.
- Compositions: To estimate the measurement from zero to the plane in case of being a large angle they used the
information of right and flat angles to facilitate the estimation. For example, if they were $270^{\circ}$, they used three right angles and converted it numerically.
- Gestures: Students simulated the segments that form angles with their hands to see if they were greater or less than the most common angles, they also used the index finger to count each line of the radar.
- Demonstration without words: Some students estimated measurements of right, flat, null and complete angles at a glance, this because the previous practices with the software made it easier for them.

Activity 4. Estimating angles in the Solar System.

- Counting: Students add and multiply the numerical quantity that represents the distance between each star to obtain the result of the angle according to the instructions given by the program.
- Gestures: by means of the expression of the face the students, showed when they did not estimate the angle measure and when they were able to estimate it.
- Demonstration without words: Students used the work in the previous activities to compare the types of angles, their measures and see if the angles shown on the screen are greater, less or equal.
Activity 5. Representing angles by positions
- Making constructions: To determine the figure according to the instructions given by the program, the students manipulate the doll's limbs until the corresponding angles appear on the screen.
Activity 6. Indicating the inclination of an angle.
- Representation systems: from the concept of angle, and according to the different activities
raised in the program, the students made mental constructions using their geometric representations, they only concentrated on firing calculating the necessary angle to shoot each barrel.

Activity 7. Measuring the angle formed by the clock hands.

Compositions: To identify the cases where the hands show angles of $90^{\circ}, 0^{\circ}, 180^{\circ}$ it was easily identified, but some of the students used compositions finding that the amplitude between each hour is 30 degrees and between each minute is $6^{\circ}$, with which they could find all the amounts.

With the program "The clock", the activities carried out and the processes that the students privilege were evidenced. Results included:

Activity 1. Assembling the clock

- Description from language: Students observed the clock hands very well and expressed that they were very different due to their color and size, reaching the conclusion that the one that indicated the hours is the thickest, shortest and yellow. The one for the minutes is a little less thick than the one for the hour, but longer and green in color. Finally, the one of the seconds that was the thinnest and the longest and its color is red.
- Transformations: When they were clear about what everyone had to do without any exception, they observed that on the right side of the screen were the numbers from 1 to 12 and the hands of the clock, and they first decided to place each number in the logical order for them from lowest to highest, from 1 and ending with 12. Most of the students placed the hour hand first, then the minute hand, and last the second hand, arguing that depending on how the time is read, the order of the hands is likewise. The other students placed the hands randomly without taking into account an order.
- Counting: Some students counted the red lines to determine what number they needed and be able to place them in order.

Activity 2. Synchronizing the time

- Demonstration without words: When the students arrived at the "set the time" box, the first thing they did was to look closely at the hands of the clock to identify the hour hand, the minute hand and the second hand. Some students singlehandedly identified them from their experiences with the watch in their daily lives. Some did notice that they were different, but could not identify them, so they asked their classmates, and even the guide, for help.
- Description from the language: they observed the hands very well and expressed that the hands were very different due to their color and size, reaching the conclusion that the one that indicated the hour is the thickest and shortest and is yellow. The one that marks the minutes is a little less thick than that of the hour, but longer and green in color. Finally, the one of the seconds that was the thinnest and the longest and its color is red.
- Representation System: some students read the instructions and observe very well the exact time indicated by the digital clock and move the hands of the clock to indicate it on the analog clock.
- Counting: In order to move the hands of the analog clock and indicate the time that the digital one showed, some students counted each line that represented the minutes and seconds, while other students counted from five by five, taking into account that the number 1 indicates 5 minutes or seconds, the 2 indicated 10 and so on until the exact time was indicated. Others only looked at the numbers that the analog clock has, and assumed that each of them were the requested hour, minutes and seconds and since they only counted up to 12. If someone went over that number, they would move the clock hand
to the number indicated in the unit, that is, if there were 14 they placed it in 4 . Some students, observing that the time went past 12 , kept counting, taking 1 as 13,2 as 14 , and so on until they reached the time indicated on the digital clock.

Activity 3. Telling the time.

- Graphs: They drew a circle on a sheet and drew a vertical and a horizontal line that divided the circle in equal parts, to determine at what number it was a quarter, a half and three quarters of the clock.
- Description from language: They expressed that this activity was very difficult for them since they did not understand how much a quarter, a half, and even the o'clock hour was.

Activity 4. Writing the time

- Transformations: the students observed very well where the hands of the clock were located to determine the hour, minutes and seconds that they should write on the digital clock. The student uses the reverse process of visualizing the activity to set the time.
- Counting: they always began to count from the number 12, since they were clear that it was the starting point, and began to count each small line to determine the minutes and seconds. To indicate the time, they counted each number until they reached the hand. Some did not count, but rather took the number into account.

Activity 5. Addition and subtraction of hours

- Transformations: When some students were given more than 60 seconds in the sum of these, they left the remaining seconds in the box and took the 60 seconds as 1 minute, adding it to their operation. When some students were given more than 60 seconds in the sum of these, they left the remaining seconds in the box and took the 60 seconds as 1 minute, adding it to their operation. The students who carried out the activity took into account the visualization of module 60.
- Description from language: Some students expressed that the subtraction was not possible when the minuend was greater than the subtrahend, since they saw it as an operation between numbers and not between hours.

In the interviews carried out with the head mathematics teachers of third and fifth grade regarding the importance of the teaching of mathematics, they stated the following:

Mathematics is fundamental in the educational process of the child, since they structure it for a logical conception of its cosmology. It is a fundamental subject, since it allows the child to develop skills, competencies, analytical skills and its application in daily life is fundamental.

Mathematics is a foundation for children's mental development, it helps them to be logical, to reason and to have a mind prepared for thought and criticism.

Mathematics creates in children a conscious and favorable disposition to take actions that lead to the solution of the problems they face every day.

Also, mathematics contributes to the formation of values in children, a way of facing logical and coherent reality, the search for accuracy in the results, a clear understanding and expression through the use of symbols.

In all grades the teaching of mathematics is important. That is, from preschool until we finish school they are important. In Elementary education it is important to highlight the work with concrete materials. Let's say that in preschool they work a lot with concrete materials, and in primary school they forget a little, but it is important for them to use concrete materials for the development of their mathematical thinking and the operations that are taught.

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Mathematics creates in children a conscious and favorable disposition to take actions that lead to the solution of the problems they face every day. Also, mathematics contributes to the formation of values in children, a way of facing logical and coherent reality, the search for accuracy in the results, a clear understanding and expression through the use of symbols.

Mathematics strengthens the logical thinking of the child from an early age, making it possible for him to later solve various problems. (Interview with teachers)

In addition, for teachers, it is very important to include math problem solving in their classes, stating that:

The problems help us to see if the student understands what type of operation to use to solve the problem.

It is very important because that is the life of children, everything in our lives moves with the solution of problems.

It is very important, since this is the basis for other subjects and for the same development of the child in their daily life, that they become familiar with different problems in their life and a way to solve them. (Interview with teachers)

Regarding the importance given to the use of ICT environments in their classes, the teachers gave the following answers:

They are very important because it helps us to diversify the methodology, the children are encouraged.

Children experience different methodologies and strategies with which more meaningful learning is achieved.

ICTs are as important as concrete materials. For some years now, people have been talking about LKT (Learning and Knowledge Technologies), forming a TPACK model (Technological Pedagogical Content Knowledge). From my concept, with these new generations, a class should be conceived with a mixture of ICTs and concrete materials, never with just one of those elements. (Interview with teachers)

Finally, to the question, how important are visualization processes in solving math problems by students? The teachers stated:

It is very important and I am going to quote Constans Cami who talks about the reinvention of the child in arithmetic and how he does that through visualization. I can also mention that for two years that we have been receiving these trainings from Japan, they have made us as an emphasis on that, in that when I am solving a problem it is best for the child to visualize it, graph it, draw it, see the quantities so that they can understand it better and solve it better. So, visualization is very important in mathematics.

It is very important. If you use the strategies from preschool years, the children will have a different learning experience. For example how we would use it, when we do the school store, so if we start with small amounts, the children will learn and you will increase the difficulty littkle by little, sure that they are going to have a significant learning. As my colleague said, right now in the training of the teachers in Japan they always tell us, that the child draw the problem, that they are asking him so that he can arrive at an answer easier. It seems to me that, if the child visualizes, draws or thinks well what they are asking, they come up with the answer easier.

It is vitally important, that is why when I was a student it was so difficult to understand algebra exercises in "Baldor's Algebra", due to the lack of real visualization or materialization as a real life problem; without this it is very difficult to understand and address it.

Students develop mental work to solve problems, and they do it there, in the course of the problem. So, it seems to me that it is very important, because they can do it with any problem and wherever they are. (Interview with teachers)

## Conclusions

In the different activities proposed in each of the participating educational institutions, it was evidenced that visualization plays a very important role in terms of students' problemsolving styles. According to the categorization of the tasks, the students privileged the following: Making graphs or sketches, constructions, synoptic diagrams, transformations, counting, folding, views, compositions, gestures, description from language, demonstrations without words and using representation systems.

With the use of ICT-mediated environments, in solving math problems with elementary school students, a greater interest in classroom work could be observed, evidenced in attitudes such as: active participation in class, development of all the activities proposed in each class, permanent search for alternative solutions to the problems raised.

The use of computer programs to solve problems generates a positive impact insofar as it allows students to immediately check the answers to them, reflect on the mistakes made and provide permanent feedback on their cognitive processes. In addition, it allows greater exercise by solving a greater number of problems compared to those solved without it.

The elementary school teachers involved in research do not normally use ICT-mediated environments in the development of mathematics classes, but by incorporating into this research process with the designed programs, they noted the need to know it and use it in classes with their students, which allowed them to recognize the importance of solving math problems focused on visualization processes, using this technological resource.

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[^0]:    1. Educational program for tablets with Android operating system, designed by Gedes (University of Quindío Software Study and Development Group).
[^1]:    2. Rural educational institution
